POST-NEWTONIAN PARAMETERS FOR GENERAL BLACK HOLE AND SPHERICALLY SYMMETRIC p-BRANE SOLUTIONS

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Black hole p-brane solutions for a wide class of intersection rules are considered. The solutions are defined on a manifold which contains a product of $(n-1)$ Ricci-flat "internal" spaces. The post-Newtonian parameters β and γ corresponding to a 4-dimensional section of the metric for general intersection rules are studied. It is shown that β does not depend but γ depends on p-brane intersections. For "block-orthogonal" intersection rules spherically symmetric solutions are considered, and explicit relations for post-Newtonian parameters are obtained. The bounds on parameters of solutions following from observational restrictions in the Solar system are presented.

1. Introduction

Multidimensional gravitational models (see, e.g., [1–5] and references therein) are rather popular at present. There exist two main approaches within the multidimensional paradigm: (**i**) extra dimensions are considered as real space-time dimensions; (**ii**) extra dimensions are considered as some imaginary (e.g. mirror, or virtual) reality, as a mathematical tool for generating the 4-metric and extra (dilatonic) scalar fields. In the second approach the 4-dimensional metric is a "real physical object" that should be extracted in some way from the multidimensional one.

In both cases we deal with exact solutions to multidimensional field equations with 4-dimensional sections which may describe some astrophysical objects (e.g. the Sun, neutron stars, etc.) and the motion of test bodies (e.g. Mercury). The 4-metric is usually considered to be a small "deformation" of the original Schwarzschild one and is governed by some parameters related to the post-Newtonian (PN) ones (for definition see Sec. 2). Applying the classical tests of general relativity: gravitational redshift, light deflection, perihelion advance and time delay, (see [6, 7]) along with the geodetic precession test, we can find the restrictions on the PN parameters.

In [8, 5] the PN parameters and classical tests were considered (in the framework of the approach (**ii**)) for a metric generalizing the Schwarzschild solution to the case of several Ricci-flat internal spaces. Later on, Kramer's more special 5-dimensional solution (also known as the Gross-Perry-Sorkin-Davidson-Owen one) was considered in [9–11] for the approach (**i**) and the classical tests (e.g. perihelion advance and geodetic

precession) were generalized to situations in which the components of momentum and spin along the extra coordinate do not vanish.

In this paper we deal with 4-dimensional sections of mutidimensional spherically symmetric and black hole solutions with generalized p-branes charged by fields of forms. In Sec. 2 black hole p-brane solutions for general intersection rules are considered and the PN parameters are calculated. In Sec. 3 more general spherically symmetric solutions are considered, but for more special "block-orthogonal" intersection rules, and explicit relations for PN parameters are obtained. In both sections the bounds on the solution gravitation parameter following ¿from observational restrictions in the Solar system are considered.

2. Black hole p-brane solutions

2.1. Solutions

We consider a model governed by the action

$$S = \int d^D x \sqrt{|g|} \Big\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{\theta_a}{n_a!} \exp[2\lambda_a(\varphi)](F^a)^2 \Big\} \quad (2.1)$$

where $g = g_{MN}(x)dx^M \otimes dx^N$ is a D-dimensional metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of scalar (dilatonic) fields, $(h_{\alpha\beta})$ is constant symmetric non-degenerate $l \times l$ matrix $(l \in \mathbb{N})$, $\theta_a = \pm 1$, $F^a = dA^a = \frac{1}{n_a!} F^a_{M_1\ldots M_{n_a}} dz^{M_1} \wedge \ldots \wedge dz^{M_{n_a}}$ is an n_a-form $(n_a \geq 1)$, λ_a is a 1-form on $\mathbb{R}^l$: $\lambda_a(\varphi) = \lambda_{\alpha a}\varphi^\alpha$, $\alpha = 1, \ldots, l$; we denote $|g| = |\det(g_{MN})|$ and $(F^a)_g^2 = F^a_{M_1\ldots M_{n_a}} F^a_{N_1\ldots N_{n_a}} g^{M_1 N_1} \cdots g^{M_{n_a} N_{n_a}}$, $a \in \Delta$. Here Δ is some finite set.

Let us consider a family of black-hole solutions to the field equations corresponding to the action (2.1).

These solutions are defined on the manifold

$$M = (R_0, +\infty) \times (M_1 = S^{d_1}) \times (M_2 = \mathbb{R}) \times \ldots \times M_n, \tag{2.2}$$

and have the following form:

$$
\begin{aligned}
g = \Big(\prod_{s \in S} H_s^{2h_s d(I_s)/(D-2)} \Big) \Big\{ & F^{-1} dR \otimes dR \\
& + R^2 d\Omega_{d_1}^2 - \Big(\prod_{s \in S} H_s^{-2h_s} \Big) F dt \otimes dt \\
& + \sum_{i=3}^{n} \Big(\prod_{s \in S} H_s^{-2h_s \delta_{iI_s}} \Big) g^i \Big\},
\end{aligned} \tag{2.3}
$$

$$\mathrm{e}^{\varphi^\alpha} = \prod_{s \in S} H_s^{h_s \chi_s \lambda_{a_s}^\alpha}, \tag{2.4}$$

$$F^a = \sum_{s \in S} \delta_{a_s}^a \mathcal{F}^s, \tag{2.5}$$

where

$$\mathcal{F}^s = \frac{Q_s}{R^{d_1}} \Big(\prod_{s' \in S} H_{s'}^{-A_{ss'}} \Big) dR \wedge \tau(I_s), \quad s \in S_e, \tag{2.6}$$

$$\mathcal{F}^s = Q_s \tau(\bar{I}_s), \qquad s \in S_m. \tag{2.7}$$

Here $Q_s \neq 0$ ($s \in S$) are charges, $R_0 > 0$, $R_0^{\bar{d}} = 2\mu > 0$, $\bar{d} = d_1 - 1$, $F = 1 - 2\mu/R^{\bar{d}}$.

In (2.3), $g^i = g^i_{m_i n_i}(y^i)$ is a Ricci-flat metric on M_i, $i = 2, \ldots, n$, and

$$\delta_{iI} = \sum_{j \in I} \delta_{ij} \tag{2.8}$$

is the indicator of i belonging to I: $\delta_{iI} = 1$ for $i \in I$ and $\delta_{iI} = 0$ otherwise. We note that here all p-branes do not "live" in M_1.

The p-brane set S is by definition

$$S = S_e \cup S_m, \quad S_v = \cup_{a \in \Delta} \{a\} \times \{v\} \times \Omega_{a,v}, \tag{2.9}$$

$v = e, m$ and $\Omega_{a,e}, \Omega_{a,m} \subset \Omega$, where $\Omega = \Omega(n)$ is the set of all non-empty subsets of $\{2, \ldots, n\}$. Thus, any $s \in S$ has the form $s = (a_s, v_s, I_s)$, where $a_s \in \Delta$, $v_s = e, m$ and $I_s \in \Omega_{a_s, v_s}$. The sets S_e and S_m define electric and magnetic p-branes, respectively. In (2.4) $\chi_s = +1, -1$ for $s \in S_e, S_m$ respectively.

All the manifolds M_i, $i > 1$ are assumed to be oriented and connected and the volume d_i-forms

$$\tau_i \equiv \sqrt{|g^i(y_i)|} \, dy_i^1 \wedge \ldots \wedge dy_i^{d_i}, \tag{2.10}$$

are well defined for all $i = 1, \ldots, n$. Here $d_i = \dim M_i$, $M_1 = S^{d_1}$, $d_1 > 1$; $D = 1 + \sum_{i=1}^{n} d_i$, and for any $I = \{i_1, \ldots, i_k\} \in \Omega$, $i_1 < \ldots < i_k$, we denote

$$\tau(I) \equiv \tau_{i_1} \wedge \ldots \wedge \tau_{i_k}, \qquad d(I) \equiv \sum_{i \in I} d_i. \tag{2.11}$$

In (2.7) $\bar{I} \equiv \{1, \ldots, n\} \setminus I$.

The parameters h_s appearing in the solution satisfy the relations

$$h_s = K_s^{-1}, \qquad K_s = B_{ss}, \tag{2.12}$$

where

$$B_{ss'} \equiv d(I_s \cap I_{s'}) + \frac{d(I_s) d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{\alpha a_s} \lambda_{\beta a_{s'}} h^{\alpha\beta}, \tag{2.13}$$

$s, s' \in S$, with $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$. Here we assume that

$$\textbf{(i)} \qquad B_{ss} \neq 0, \tag{2.14}$$

for all $s \in S$, and

$$\textbf{(ii)} \qquad \det(B_{ss'}) \neq 0, \tag{2.15}$$

i.e., the matrix $(B_{ss'})$ is non-degenerate. Let

$$(A_{ss'}) = (2B_{ss'}/B_{s's'}). \tag{2.16}$$

Here some ordering in S is assumed.

The functions $H_s = H_s(z) > 0$, $z = 2\mu/R^{\bar{d}} \in (0, 1)$ obey the equations

$$\frac{d}{dz} \left(\frac{(1-z)}{H_s} \frac{d}{dz} H_s \right) = B_s \prod_{s' \in S} H_{s'}^{-A_{ss'}}, \tag{2.17}$$

equipped with the boundary conditions

$$H_s(1 - 0) = H_{s0} \in (0, +\infty), \tag{2.18}$$

$$H_s(+0) = 1, \tag{2.19}$$

$$B_s = K_s \varepsilon_s Q_s^2 / (2\mu)^2, \tag{2.20}$$

$s \in S$. Here

$$\varepsilon_s = (-\varepsilon[g])^{(1-\chi_s)/2} \varepsilon(I_s) \theta_{a_s}, \tag{2.21}$$

$s \in S$, $\varepsilon[g] \equiv \operatorname{sign} \det(g_{MN})$. More explicitly, (2.21) reads: $\varepsilon_s = \varepsilon(I_s) \theta_{a_s}$ for $v_s = e$ and $\varepsilon_s = -\varepsilon[g] \varepsilon(I_s) \theta_{a_s}$, for $v_s = m$.

Eqs. (2.17) are equivalent to special Toda-type equations. The first boundary condition (2.18) guarantees the existence of a horizon at $R^{\bar{d}} = 2\mu$ for the metric (2.3). The second condition (2.19) ensures the asymptotic (at $R \to +\infty$) flatness of the $(2 + d_1)$-section of the metric.

Due to (2.6) and (2.7), the p-brane worldsheet dimension $d(I_s)$ is defined by

$$d(I_s) = n_{a_s} - 1, \quad d(I_s) = D - n_{a_s} - 1, \tag{2.22}$$

for $s \in S_e, S_m$ respectively. For a p-brane: $p = p_s = d(I_s) - 1$.

The solutions are valid if the following restrictions on the sets $\Omega_{a,v}$ are imposed. (These restrictions guarantees the block-diagonal structure of the stress-energy tensor, as for the metric, and the existence of a σ-model representation [13]). We denote $w_1 \equiv \{i | i \in \{1, \ldots, n\}, \quad d_i = 1\}$, and $n_1 = |w_1|$ (i.e. n_1 is the number of 1-dimensional spaces among M_i, $i = 1, \ldots, n$).

Restriction . Let 1a) $n_1 \leq 1$ or 1b) $n_1 \geq 2$ and for any $a \in \Delta$, $v \in \{e, m\}$, $i, j \in w_1$, $i < j$, there are no $I, J \in \Omega_{a,v}$ such that $i \in I$, $j \in J$ and $I \setminus \{i\} = J \setminus \{j\}$.

This restriction is satisfied in the non-composite case: $|\Omega_{a,v}| = 1$, (i.e. when there is only one p-brane for each value of the colour index a, $a \in \Delta$); The restriction forbids certain intersections of two p-branes with the same colour index for $n_1 \geq 2$.

The Hawking "temperature" corresponding to the solution is found to be

$$T_H = \frac{\bar{d}}{4\pi(2\mu)^{1/\bar{d}}} \prod_{s \in S} H_{s0}^{-h_s}, \tag{2.23}$$

where H_{s0} are defined in (2.18)

"Block-orthogonal" black holes [16]. Let

$$S = S_1 \sqcup \ldots \sqcup S_k, \tag{2.24}$$

$S_i \neq \emptyset$, $i = 1, \ldots, k$, and

(iii) $(U^s, U^{s'}) = 0 \tag{2.25}$

for all $s \in S_i$, $s' \in S_j$, $i \neq j$; $i, j = 1, \ldots, k$. In [14, 15] "block-orthogonal" solutions were obtained:

$$H_s(z) = (1 + \bar{P}_s z)^{b_0^s}, \tag{2.26}$$

$s \in S$, where

$$b_0^s = 2 \sum_{s' \in S} A^{ss'}, \tag{2.27}$$

$((A^{ss'}) = (A_{ss'})^{-1})$ and $\bar{P}_s = \bar{P}_{s'}$, $s, s' \in S_i$, $i = 1, \ldots, k$.

Let $(A_{ss'})$ be a Cartan matrix for a finite-dimensional semisimple Lie algebra $\mathcal{G}$. In this case all powers in (2.26) are positive integers [17]:

$$b_0^s = n_s \in \mathbb{N}, \tag{2.28}$$

and hence all functions H_s are polynomials, $s \in S$.

Conjecture. *Let $(A_{ss'})$ be a Cartan matrix for a semisimple finite-dimensional Lie algebra $\mathcal{G}$. Then the solutions of Eqs. (2.17)–(2.19) are polynomials*

$$H_s(z) = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k, \tag{2.29}$$

where $P_s^{(k)}$ are constants, $k = 1, \ldots, n_s$, $P_s^{(n_s)} \neq 0$, $s \in S$.

In [12] this Conjecture was verified for simple Lie algebras $A_m = sl(m + 1, \mathbb{C})$, $m \geq 1$. In extremal case ($\mu = +0$) an a analogue of this conjecture was suggested previously in [18].

2.2. Post-Newtonian approximation

Let $d_1 = 2$. Here we consider the 4-dimensional section of the metric (2.3), namely,

$$g^{(4)} = U \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 d\Omega_2^2 - U_1 \left(1 - \frac{2\mu}{R}\right) dt \otimes dt \right\} \tag{2.30}$$

in the range $R > 2\mu$, where

$$U = \prod_{s \in S} H_s^{2d(I_s) h_s / (D-2)}, \tag{2.31}$$

$$U_1 = \prod_{s \in S} H_s^{-2h_s}. \tag{2.32}$$

Let us imagine that some real astrophysical objects (e.g. stars) may be described (or approximated) by the 4-dimensional physical metric (2.30), i.e. they are traces of extended multidimensional objects (charged p-brane black holes).

In the post-Newtonian approximation we restrict ourselves to the first two powers of $1/R$, i. e.

$$H_s = 1 + \frac{P_s}{R} + \frac{P_s^{(2)}}{R^2} + o\left(\frac{1}{R^3}\right), \tag{2.33}$$

for $R \to +\infty$, $s \in S$.

Introducing a new radial variable ρ by the relation

$$R = \rho \left(1 + \frac{\mu}{2\rho}\right)^2, \tag{2.34}$$

($\rho > \mu/2$), we may rewrite the metric (2.30) in a 3-dimensional conformally flat form and calculate the post-Newtonian parameters β and γ (Eddington parameters) using the following standard relations:

$$g_{00}^{(4)} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{2.35}$$

$$g_{ij}^{(4)} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{2.36}$$

$i, j = 1, 2, 3$, where $V = GM/\rho$ is Newton's potential, G is the gravitational constant and M is the gravitational mass. From (2.35)-(2.36) we deduce the formulae

$$GM = \mu + \sum_{s \in S} h_s P_s \left(1 - \frac{d(I_s)}{D-2}\right) \tag{2.37}$$

and

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} h_s (P_s^2 + 2\mu P_s$$
$$- 2P_s^{(2)}) \left(1 - \frac{d(I_s)}{D-2}\right), \tag{2.38}$$

$$\gamma - 1 = -\frac{1}{GM} \sum_{s \in S} h_s P_s \left(1 - 2\frac{d(I_s)}{D-2}\right). \tag{2.39}$$

For special "block-orthogonal" solutions this relation coincides with that of [16]. For a more general

spherically symmetric case it was obtained in [15] (see next section).

The parameter β is determined by the squared charges Q_s^2 of the p-branes (or, more correctly, by the charge densities) and the signature parameters ε_s.

Now, we will show that in the general case, as in [16], the following theorem is valid:

Theorem. *The parameter β does not depend on the dimensions of p-brane intersections.*

Proof. ¿From (2.17) we get in the zero order of a z-decomposition:

$$P_s^2 + 2\mu P_s - 2P_s^{(2)} = -K_s \varepsilon_s Q_s^2, \qquad (2.40)$$

$s \in S$. Hence,

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} (-\varepsilon_s) Q_s^2 \left(1 - \frac{d(I_s)}{D-2}\right). \quad (2.41)$$

Thus β depends on Q_s/GM, i.e., on the ratios of the charges and the mass. It does not depend on the intersections. The theorem is proved.

The parameter β is obtained without knowledge of the general solution for H_s and does not depend on the quasi-Cartan matrix and hence on p-brane intersections. The parameter γ depends on the ratios P_s/GM, where P_s are functions of GM, Q_s and also $A = (A_{ss'})$. Thus it depends on the intersections. The calculation of γ needs an exact solution for the radial functions H_s.

For the most physically interesting p-brane solutions, $\varepsilon_s = -1$ and $d(I_s) < D - 2$ for all $s \in S$, whichimplies

$$\beta > 1. \qquad (2.42)$$

Using the relations (2.39) and (2.41), we can obtain bounds on β and γ which may be compared with the experimental data.

Observational restrictions. The observations in the Solar system give tight constraints on the Eddington parameters [7]:

$$\gamma = 1.000 \pm 0.002 \qquad (2.43)$$
$$\beta = 0.9998 \pm 0.0006. \qquad (2.44)$$

The first restriction is a result of the Viking time-delay experiment [19]. The second one follows from (2.43) and the analysis of the lunar laser ranging data. In this case a high precision test based on the calculation of the combination $(4\beta - \gamma - 3)$, appearing in the Nordtvedt effect [21], is used [20].

For small enough $p_s = P_s/GM$ and $P_s^{(2)} \sim P_s^2$ we get $GM \sim \mu$ and hence

$$\beta - 1 \sim \sum_{s \in S} h_s p_s \left(1 - \frac{d(I_s)}{D-2}\right), \qquad (2.45)$$

$$\gamma - 1 \sim -\sum_{s \in S} h_s p_s \left(1 - 2\frac{d(I_s)}{D-2}\right), \qquad (2.46)$$

i.e., $\beta - 1$ and $\gamma - 1$ are of the same order. Thus, for small enough p_s, it is possible to fit the Solar-system restrictions (2.43) and (2.44).

3. Spherically symmetric solutions in the block-orthogonal case

3.1. Solutions

Here we consider the spherically symmetric solutions in the block-orthogonal case (2.24)–(2.25).

The solutions for the metric and scalar fields may be written as follows:

$$g = \left(\prod_{s \in S} \bar{H}_s^{2\eta_s d(I_s)\nu_s^2/(D-2)}\right)\left\{F^{b^1-1} dR \otimes dR \right.$$
$$+ R^2 F^{b^1} d\Omega_{d_1}^2 - \left(\prod_{s \in S} \bar{H}_s^{-2\eta_s \nu_s^2}\right) F^{b^1} dt \otimes dt$$
$$\left. + \sum_{i=3}^{n} \left(\prod_{s \in S} \bar{H}_s^{-2\eta_s \nu_s^2 \delta_{iI_s}}\right) F^{b^i} g^i\right\}, \qquad (3.1)$$

$$\varphi^\alpha = \sum_{s \in S} \eta_s \nu_s^2 \chi_s \lambda_{a_s}^\alpha \ln \bar{H}_s + \frac{1}{2} b^\alpha \ln F, \qquad (3.2)$$

where $F = 1 - 2\mu/R^{\bar{d}}$,

$$\bar{H}_s = \hat{H}_s F^{(1-b_s)/2}, \qquad (3.3)$$

$$\hat{H}_s = 1 + \hat{P}_s \frac{(1 - F^{b_s})}{2\mu b_s}, \qquad (3.4)$$

$s \in S$. Here

$$\sum_{s' \in S} (U^s, U^{s'}) \eta_{s'} \nu_{s'}^2 = 1 \qquad (3.5)$$

for all $s \in S$. The parameters b_s and $\hat{P}_s$ coincide within the blocks:

$$b_s = b_{s'}, \qquad \hat{P}_s = \hat{P}_{s'}, \qquad (3.6)$$

$s, s' \in S_i$, $i = 1, \ldots, k$. The parameters b_s, b^i, b^α obey the relations

$$U^1(b) = -b^1 + \sum_{j=1}^{n} d_j b^j = 1, \qquad (3.7)$$

$$U^s(b) = \sum_{i \in I_s} d_i b^i - \chi_s \lambda_{a_s \alpha} b^\alpha = 1, \qquad (3.8)$$

$s \in S$, and

$$\sum_{s \in S} \eta_s \nu_s^2 (b_s^2 - 1) + h_{\alpha\beta} b^\alpha b^\beta + \sum_{i=2}^{n} d_i (b^i)^2$$
$$+ \frac{1}{d_1 - 1}\left(\sum_{i=2}^{n} d_i b^i\right)^2 = \frac{d_1}{d_1 - 1}. \qquad (3.9)$$

The fields of forms in (2.5) are given by the relations

$$\mathcal{F}^s = d\Phi^s \wedge \tau(I_s), \qquad (3.10)$$

$$\mathcal{F}^s = \exp[-2\lambda_{a_s}(\varphi)] * (d\Phi^s \wedge \tau(I_s)), \qquad (3.11)$$

where $* = *[g]$ is the Hodge operator and

$$\Phi^s = \nu_s/H_s', \tag{3.12}$$

$$H_s' = \left[1 - P_s'\hat{H}_s^{-1}\frac{(1-F^{b_s})}{2\mu b_s}\right]^{-1}, \tag{3.13}$$

$s \in S$. Here

$$(P_s')^2 = -\varepsilon_s\eta_s\hat{P}_s(\hat{P}_s + 2b_s\mu), \tag{3.14}$$

$s \in S$.

3.2. Post-Newtonian parameters

Let $d_1 = 2$. Consider the 4-dimensional section of the metric (3.1)

$$g^{(4)} = U\left\{F^{b^1-1}dR \otimes dR + F^{b^1}R^2 d\hat{\Omega}_2^2 - U_2 F^{b^2} dt \otimes dt\right\} \tag{3.15}$$

where $F = 1 - 2\mu/R$ and

$$U = \prod_{s \in S}\bar{H}_s^{2\eta_s d(I_s)\nu_s^2/(D-2)}, \tag{3.16}$$

$$U_2 = \prod_{s \in S}\bar{H}_s^{-2\eta_s\nu_s^2}, \tag{3.17}$$

$R > 2\mu$.

Introducing the radial variable ρ by the relation (2.34), we rewrite the metric (3.15) in a 3-dimensional conformally-flat form:

$$g^{(4)} = U\left\{-U_2 F^{b^2}dt \otimes dt \right.$$
$$\left. + F^{b^1}\left(1 + \frac{\mu}{2\rho}\right)^4 \delta_{ij}dx^i \otimes dx^j\right\}, \tag{3.18}$$

$$F = \left(1 - \frac{\mu}{2\rho}\right)^2\left(1 + \frac{\mu}{2\rho}\right)^{-2}, \tag{3.19}$$

where $\rho^2 = |x|^2 = \delta_{ij}x^ix^j$ $(i,j = 1,2,3)$.

From (3.18)–(3.19) and (2.35)–(2.36) we get for the gravitational mass:

$$GM = \mu b^2$$
$$+ \sum_{s \in S}\eta_s\nu_s^2[\hat{P}_s + (b_s - 1)\mu]\left(1 - \frac{d(I_s)}{D-2}\right) \tag{3.20}$$

and, for $GM \neq 0$,

$$\beta - 1 = \frac{1}{2(GM)^2}\sum_{s \in S}\eta_s\nu_s^2\hat{P}_s(\hat{P}_s + 2b_s\mu)\left(1 - \frac{d(I_s)}{D-2}\right), \tag{3.21}$$

$$\gamma - 1 = -\frac{1}{GM}\left[\mu(b^1 + b^2 - 1) \right.$$
$$\left. + \sum_{s \in S}\eta_s\nu_s^2[\hat{P}_s + (b_s - 1)\mu]\left(1 - 2\frac{d(I_s)}{D-2}\right)\right]. \tag{3.22}$$

As follows from (3.14), (3.21) and the inequalities $d(I_s) < D - 2$ (for all $s \in S$),

$$\beta > 1 \quad \text{if all} \quad \varepsilon_s = -1, \tag{3.23}$$

$$\beta < 1 \quad \text{if all} \quad \varepsilon_s = +1. \tag{3.24}$$

There exists a large variety of configurations with $\beta = 1$ when the relation $\varepsilon_s = $ const is broken.

There also exist non-trivial p-brane configurations with $\gamma = 1$.

Proposition. *Let the set of p-branes consist of several pairs of electric and magnetic branes. Let any such pair $(s, \bar{s} \in S)$ correspond to the same colour index, i.e. $a_s = a_{\bar{s}}$, and $\hat{P}_s = \hat{P}_{\bar{s}}$, $b_s = b_{\bar{s}}$, $\eta_s\nu_s^2 = \eta_{\bar{s}}\nu_{\bar{s}}^2$. Then for $b^1 + b^2 = 1$ we get*

$$\gamma = 1. \tag{3.25}$$

The Proposition can be readily proved using the relation $d(I_s) + d(I_{\bar{s}}) = D - 2$, following from (2.22).

For small enough $\hat{p}_s = \hat{P}_s/GM$, $b_s - 1$, $b^2 - 1$, b^i $(i > 2)$ of the same order we get $GM \sim \mu$ and hence

$$\beta - 1 \sim \sum_{s \in S}\eta_s\nu_s^2\hat{p}_s\left(1 - \frac{d(I_s)}{D-2}\right) \tag{3.26}$$

$$\gamma - 1 \sim -b^1 - b^2 + 1$$
$$- \sum_{s \in S}\eta_s\nu_s^2[\hat{p}_s + (b_s - 1)]\left(1 - 2\frac{d(I_s)}{D-2}\right), \tag{3.27}$$

i.e., $\beta - 1$ and $\gamma - 1$ are of the same order. Thus for small enough $\hat{p}_s$, $b_s - 1$, $b^2 - 1$, b^i $(i > 2)$ it is possible to fit the Solar-system restrictions (2.43) and (2.44).

There also exists another opportunity to satisfy these restrictions.

One-brane case. Consider the special case of a single p-brane. In this case we have

$$\eta_s\nu_s^{-2} = d(I_s)\left(1 - \frac{d(I_s)}{D-2}\right) + \lambda^2. \tag{3.28}$$

The relations (3.21), (3.22) and (3.28) imply that for large enough values of the dilatonic coupling constant squared λ^2 and $b^1 + b^2 = 1$ it is possible to "fine-tune" the parameters (β, γ) near the point $(1, 1)$ even if the parameters $\hat{P}_s$ are large.

4. Conclusions

We have considered a family of black-hole solutions from [12] with intersecting p-branes with next to arbitrary intersection rules. The metric of the solutions contains $n - 1$ Ricci-flat "internal" space metrics. The solutions are defined up to a set of functions H_s obeying a set of equations with certain boundary conditions. In [12] a conjecture on the polynomial structure of H_s for intersections related to semisimple Lie algebras was suggested and proved for the A_m series.

Here we have presented the post-Newtonian parameters β and γ corresponding to the 4-dimensional section of the metric. The parameter β is written in terms of ratios of the physical parameters: the charges Q_s and the mass M. Thus β does not depend on p-brane intersections. Unlike that, the parameter γ is intersection-dependent, and its calculation requires an exact solution for the radial functions H_s. We have considered the post-Newtonian parameters for spherically symmetric solutions in the block-orthogonal case and singled out the "corridors" for the b-parameters which follow ¿from the Solar-system observational restrictions.

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References

[1] V. De Sabbata and E. Schmutzer, "Unified Field Theories in More Than Four Dimensions", World Scientific, Singapore, 1982.

[2] H.C. Lee, "An Introduction to Kaluza-Klein Theories", World Scientific, Singapore, 1984.

[3] Yu.S. Vladimirov, "Physical Space-Time Dimension and Unification of Interactions", Moscow State University Press, 1987 (in Russian).

[4] P.S. Wesson and J. Ponce de Leon, *Gen. Rel. Gravit.* **26**, 555 (1994).

[5] V.N. Melnikov, "Multidimensional Classical and Quantum Cosmology and Gravitation. Exact Solutions and Variations of Constants", CBPF-NF-051/93, Rio de Janeiro, 1993, 93pp.
V.N. Melnikov, "Multidimensional Cosmology and Gravitation". CBPF-MO-002/95, Rio de Janeiro, 1995, 210pp.

[6] C.M. Will, "Theory and Experiment in Gravitational Physics", Cambridge University Press, Cambridge, 1993.

[7] T. Damour, "Gravitation, experiment and cosmology", gr-qc/9606079.

[8] K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, "On relativistic tests in multidimensional gravity", 1992 (unpublished).

[9] P.H. Lim, J.M. Overduin and P.S. Wesson, *J. Math. Phys.*, **36**, 6907 (1995).

[10] D. Kalligas, P.S. Wesson and C.W.F. Everitt, *Astrophys. J.*, **439**, 548 (1995).

[11] H. Liu and J. M. Overduin, "Solar system tests of higher-dimensional gravity", hep-th/0002132.

[12] V.D. Ivashchuk and V.N. Melnikov, "Black-hole p-brane solutions for general intersection rules", *Grav & Cosmol.*, **6** 1 (21), 27 (2000), hep-th/9910041;
"Toda p-brane black holes and polynomials related to Lie algebras", To appear in *Class. Quantum Grav.*, 2000; math-ph/0002048.

[13] V.D. Ivashchuk and V.N. Melnikov, "Sigma-model for generalized composite p-branes", hep-th/9705036; *Class. Quantum Grav.* **14**, 3001 (1997); Corrigenda *ibid.* **15** (12), 3941 (1998).

[14] K.A. Bronnikov, "Block-orthogonal brane systems, black holes and wormholes", *Grav. & Cosmol.*, **4**, 49 (1998); hep-th/9710207.

[15] V.D. Ivashchuk and V.N. Melnikov, "Multidimensional cosmological and spherically symmetric solutions with intersecting p-branes", gr-qc/9901001;
In: Lecture Notes in Physics, v. 537, Mathematical and Quantum Aspects of Relativity and Cosmology. Eds.: S.Cotsakis and G.Gibbons. Springer, Berlin, 2000, p.214.
J. Math. Phys. **40**, No 12, 6568 (1999).

[16] S. Cotsakis, V.D. Ivashchuk and V.N. Melnikov, "p-Brane black holes and post-Newtonian approximation", *Grav. & Cosmol.* **5**, No 1 (17), 52 (1999); gr-qc/9902148.

[17] M.A. Grebeniuk and V.D. Ivashchuk, Algebras", *Phys. Lett.* **B 442**, 125 (1998); hep-th/9805113.

[18] H. Lü, J. Maharana, S. Mukherji and C.N. Pope, "Cosmological solutions, p-branes and the Wheeler-DeWitt equation", hep-th/9707182.

[19] R.D. Reasenberg et. al., *Astrophys. J.* **234**, L219 (1979).

[20] J.O. Dickey et al., *Science* **265**, 482 (1994).

[21] K. Nordtvedt, *Phys. Rev.* **169**, 1017 (1968).